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THE GRAETZ PROBLEM

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RECENTLY Hwang and Yih [1] noted that it is inaccurate to use the asymptotic solution [2] of the laminar convection ("Graetz") problem for the boundary condition of the third kind when the Biot number is small. Especially, the error is prohibitive for the boundary condition of the second kind, zero Biot number.

The above conclusion is evidently correct if only the first terms of the asymptotic series are used, as in [2]. If the second terms are included, the accuracy for all values of the Biot number (*Bi*) is considerably increased as explained previously [3].

Using the results of [3], the eigenvalues for the boundary condition of the third kind is, in case of laminar flow in a circular tube, obtained from the equation

$$\frac{aK_1 \sin\left((\sqrt{\lambda})\dfrac{\pi}{2} + \dfrac{\pi}{3} - \delta\right)}{2bK_1'\lambda^{\frac{1}{3}} \sin\left((\sqrt{\lambda})\dfrac{\pi}{2} - \dfrac{\pi}{3} + \delta'\right)} = \frac{1}{Bi}$$

which corresponds to equation (2) of [1]. Here λ is the eigenvalue (defined as $\lambda^2/4$ of [1]). The other symbols are explained in [3].

The eigenvalue equation must be solved by numerical methods. Since the correction factors K, K' are nearly unity and δ, δ' are small angles, the solution is not difficult. In fact, the limiting cases of zero and infinite Biot numbers have been solved by using a slide rule only. Table 1 is a comparison of the eigenvalues (λ as defined in [3] and in this note) obtained from the three references cited:

Table 1

	$Bi = 0$			$Bi = \infty$		
Reference	[1]	[2]	[3]	[1]	[2]	[3]
$\sqrt{\lambda_0}$	0·002427	0·667	0	—	—	—
$\sqrt{\lambda_1}$	2·534	2·667	2·531	1·352	1·333	1·351
$\sqrt{\lambda_2}$	4·579	4·667	4·578	3·340	3·333	3·340
$\sqrt{\lambda_3}$	6·60	6·667	6·599	5·335	5·333	5·337
.		.	.		.	.
.		.	.		.	.
.		.	.		.	.
$\sqrt{\lambda_{10}}$		20·667	20·636		19·333	19·334

It is not clear how [1] obtained a non-vanishing λ_0 for $Bi = 0$. The solution $\lambda_0 = 0$ is elementary.

The cases of turbulent flow and for flat duct are easily included [3, 4].

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